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Theory of hypersonic surface waves in solids

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The propagation of hypersonic surface waves in solids is discussed in connection with variations of the force constants and density in the atomic boundary layer and with regard for surface tension and the influence of the lattice. The results can be utilized in hypersonic investigations of the physical properties of the surfaces of solids.

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With further increases in the frequency of surface waves propagating in a solid the influence of the atomic boundary layer begins to manifest itself and can be utilized to investigate the physical properties of surfaces, the surface tension in particular. It is well known, however, that the presence of an interface in the general case is characterized not only by surface tension, but also by variations of the force constants and density in the surface layer; these variations can be appreciable in the case of certain crystals. Moreover, at very high frequencies (>100 GHz or higher) the discreteness of the crystal structure must be taken into consideration. We propose to analyze the influence of the indicated factors on the properties of surface waves.

We assume at the outset that the surface wavelength is large enough that the influence of the lattice can be neglected. Interpreting surface effects in the Gibbssense (Ref. 4), we characterize the variations of the force constants in the boundary layer by means of a surface-tension tensor that depends on the surface strains:

$$\gamma_{ij} = \gamma_{ij}^{0} + c_{ij}^{s} \varepsilon_{s},$$

where $\gamma_{ij}^{0}$ is the known (unperturbed) surface-tension tensor, $c_{ij}^{s}$ denotes the surface elastic moduli, which are defined by the expression

$$c_{ij}^{s} = \int_{x} \left[ \varepsilon_{ij}^{s}(x_{s}) - c_{ij}^{s} \right] d x_{s},$$

$\varepsilon_{ij}(x_{s})$ denotes the bulk moduli depending on the normal coordinate $x_{s}$ due to the presence of the surface layer, $c_{ij}^{s}$ denotes the bulk moduli in the depth of the crystal, and $\varepsilon_{s}$ denotes the surface strains (the indices $i, j, m, n$, and $s$ take the surface values $t$ and $\tau$). We similarly define the surface density $\rho_{s}$. Clearly, $c_{ij}^{s}$ and $\rho_{s}$ can take positive and negative values. Proceeding from this point as in Ref. 1, we obtain the following boundary conditions for the time-harmonic fields generalizing the conditions of Ref. 1:

$$a_{i} - \frac{\partial v_{i}}{\partial z} - \alpha \rho a_{s} = 0,$$

$$a_{m} + \gamma_{m} \left( \frac{1}{R_1} + \frac{1}{R_2} \right) - \alpha \rho a_{s} = 0,$$

where $R_1$ and $R_2$ are the principal radii of curvature of the surface and $n$ is the normal coordinate index. The equations of motion in Ref. 1 remain unaltered here. Continuing the ensuing discussion to the case of an isotropic boundary, i.e., setting $\gamma_{ij}(t) = \gamma_{ij}^{0} + \lambda \delta_{ij} \varepsilon_{r}$, and

$$2 \mu \varepsilon_{i},$$

where $\gamma$ is the ordinary surface tension, we analyze the propagation of a Rayleigh wave along the $x_{t}$ axis. It is readily shown that the presence of the additional terms in Eq. (1) over and above those in Ref. 1 induces additional Rayleigh-wave dispersion, and the approximate expression for the phase velocity takes the form

$$c_{r} = \frac{\eta_{s}^{N} \left[ 1 + \frac{\dot{\rho}_{0}}{\eta_{s}} \left( \frac{\gamma}{\mu} + g \lambda - \mu \right) \right]}{\mu \rho},$$

where $c_{r}$ is the transverse wave velocity, $\eta_{s}$ is the root of the Rayleigh equation, and $\mu, \gamma, \lambda, s \sim 1$ are constants depending on the elastic properties of the medium. An expression for $p$ is given in Ref. 1, and $g$ and $s$ are given by the equations $g = (1 - \gamma_{0})^{1/2}(1 - \delta \gamma_{0})^{-1/2}$ and $s = \gamma_{0}(1 + g)$, where $\theta = c_{r}^{2}/c_{s}^{2}$. Estimates of the quantities $\lambda_{s}$ and $\rho_{s}$ for a number of cubic crystals show that their combined contribution to the variation of the velocity can amount to 40% of the contribution of $\gamma$.

An intriguing feature associated with the introduction of the strain-dependent surface tension and surface density is the possibility of the existence of a pure shear surface acoustic wave. Thus, assuming that the displacement is directed along the $x_{r}$ axis, we deduce the following
boundary condition from (1): \(
\sigma_{\text{tr}} = \mu^S \partial^2 u_{\text{tr}} / \partial x^2 + \omega^2 \rho^S u_{\text{tr}} = 0,
\)
whereupon for a wave traveling along the \(x_1\) axis and decaying slowly into the depth according to an exponential law with exponent \(q = (k^2 - k_0^2)^{1/2}\) we obtain the dispersion relation
\[
\frac{c}{c_0} = 1 - \frac{\omega^2}{2c_0^2} \left( \frac{\rho^0}{\rho^S} \right) \left( \frac{\mu^0}{\mu^S} \right)^2.
\]
(3)

At a frequency of 30 GHz the localization depth \(l \sim q^{-1}\) is \(\sim 100 \mu\). This wave, which is of definite interest for high-frequency surface investigations, does not differ in principle from waves of the Love type. All that differs is the particular method of analysis, within the scope of which we were not previously concerned with the structure of the field in the surface layer per se.

As the spatial period of the field approaches the interatomic spacing it is necessary to consider the influence of the crystal lattice; this can be done by means of the nonlocal theory of elasticity,\(^7\)\(^1\) in which the classical elastic moduli are replaced by corresponding operators. We use an elementary model proposed\(^7\) for calculation of the dispersion of Rayleigh waves, where the nonlocal character of the medium is considered only in the direction tangential to the surface. Here, of course, information is lost about the detailed structure of the field in the normal direction. We recall, however, that this is precisely the approximation in which the integral parameters used above, i.e., the surface tension and surface elasticity and density, are meaningful. Proceeding as in Ref. 7, we can easily show that the expressions describing the phase velocities of Rayleigh and shear surface waves within the framework of the nonlocal theory take the forms (2) and (3), in which the quantities \(\mu, \mu^S, \) and \(\lambda^S\), which enter into (2) and (3) both explicitly and in terms of \(q_0\), must be replaced by the Fourier transforms of the kernels of the integral operators \(\{ (k^2 \mu^S) / \sin k^2 d/2, \}
\)
(4/\(k^2 \mu^S / \sin k^2 d/2\), obtained by comparison of the nonlocal theory with the theory of lattice dynamics. Here, \(\mu, \mu^S, \) and \(\lambda^S\) are the ordinary local moduli, and \(d\) is the lattice constant. It is seen at once that with an increase in \(kd\) the discreteness of the medium magnifies the effect of the boundary layer on the surface-wave phase velocities.

In the case of anomalous dispersion it can turn out that Rayleigh waves are nonexistent in general at frequencies close to the limit, i.e., \(\sim 10^{12}\) Hz, where, according to (2), the inequality \(c > c_0\) holds. The Rayleigh-wave energy is radiated into the volume of the medium in this event.

While the present article was in press we learned of the work of Murdoch\(^8\) and the related work of Velasco and Garcia-Molinier,\(^9\) who have used an elastic membrane model to analyze the influence of surface effects on the propagation of surface acoustic waves, including Rayleigh and Stoneley waves. In particular, they mention the possibility of the existence of pure shear surface waves in crystals. However, the corresponding dispersion relation and a number of other results of these investigations are in doubt because of what, in our opinion, is an incorrect determination of the surface elasticity and surface density from the membrane model, in contradiction with the Gibbs interpretation of the surface increment of a physical quantity.

Modern techniques for the excitation of coherent and thermal phonons in the terahertz range are discussed in Ref. 10.

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